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# Taylor's hypothesis and the probability density functions of temporal velocity and temperature derivatives in a turbulent flow

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Equations for the instantaneous velocity and temperature fluctuations in a turbulent flow are used to assess the effect of a fluctuating convection velocity on Taylor's hypothesis when certain simplifying assumptions are made. The probability density function of the velocity or temperature derivative is calculated, with an assumed Gaussian probability density function of the spatial derivative, for two cases of the fluctuating convection velocity. In the first case, the convection velocity is the instantaneous longitudinal velocity, assumed to be Gaussian. In the second, the magnitude of the convection velocity is equal to that of the total velocity vector whose components are Gaussian. The calculated probability density function shows a significant departure, in both cases, from the Gaussian distribution for relatively large amplitudes of the derivative, at only moderate values of the turbulence intensity level. The fluctuating convection velocity affects normalized moments of measured velocity and temperature derivatives in the atmospheric surface layer. The effect increases with increasing order of the moment and is more significant for odd-order moments than even-order moments.

## 1. Introduction

The frozen field or Taylor's hypothesis allows temporal fluctuations to be interpreted as convected streamwise spatial fluctuations. It asserts that

$$u_i(\mathbf{x}, t) = u_i(\mathbf{x} - \mathbf{U}\tau, t+\tau)$$

for not too large values of the time delay $\tau$. Two important applications of this approximation (see Townsend 1976) are the calculation of one-dimensional spectrum functions from measured frequency spectra and the determination of mean values of functions of spatial turbulent velocity (or scalar) gradients from measured temporal derivative fluctuations via the often used relation

$$\frac{\partial}{\partial t} = -U_1 \frac{\partial}{\partial x_1}, \tag{1}$$

where $U_1$ is the mean velocity in the $x_1$ direction. Even when the convection velocity of the large-scale velocity pattern differs considerably from $U_1$, it is thought that (1) is reasonably accurate, at least when the turbulence intensity $\sigma_{u_1}/U_1$ is small, as small eddies are convected with the mean velocity of the fluid.

Fisher & Davies (1964) suggested several mechanisms which can prevent the direct interpretation of spectra as space spectra by Taylor's hypothesis. Lumley (1965) showed that only one of these mechanisms, namely the convection velocity fluctuation, remained important at high frequencies. Lumley proposed a model in which the characteristic function of convection velocity fluctuation was represented by the first two terms of its series expansion. The model led to a formula for correcting the high-frequency part of the one-dimensional streamwise velocity spectrum. Wyngaard & Clifford (1977) considered a Gaussian convection velocity fluctuation and extended Lumley's model† to spectra of lateral velocity and scalar fluctuations. They found that, while the inertial subrange spectrum was not significantly affected, larger wave-numbers could suffer serious spectral distortion. Champagne (1978) obtained an analytical solution to Lumley's spectral correction formula and found that, for jet data ($R_\lambda = 626$), the measured spectrum overestimates the correct spectrum by 200 % at frequencies near the Kolmogorov frequency. This sort of error underlines the need to correct the high-frequency part of the spectrum before an assessment can be made, as was done by Champagne, of the effect of turbulence Reynolds number $R_\lambda$ ($= \sigma_{u_1}\lambda/\nu$, where $\lambda$ is the Taylor microscale) on the spectral shape. As a result of Kolmogorov's third hypothesis the shape of the probability density function of $\partial u/\partial x$ or $\partial\theta/\partial x$ ($\theta$ is the temperature fluctuation) should exhibit a significant dependence (e.g. Wyngaard & Pao 1972) on $R_\lambda$. It would therefore seem important to evaluate the possible effect of a fluctuating convection velocity on the shape of the probability density function of the spatial velocity or temperature derivative. No such direct evaluation appears to have been made, but relations between moments of $u_{1,t}$ ($\equiv \partial u_1/\partial t$) and $u_{1,1}$ ($\equiv \partial u_1/\partial x_1$) have been obtained by Lumley, Wyngaard & Pao and Wyngaard & Clifford via the corrected spectrum. These relations are consistent with those obtained by Heskestad (1965) from a consideration of the Navier–Stokes equations under certain simplifying assumptions. (Both the spectral model of Lumley and the approach of Heskestad assume that the convection velocity fluctuation is independent of the convected small-scale structure.) Wyngaard & Tennekes (1970) found that, while second-order and third-order moments of $(u_{1,t}/U_1)$ in a curved mixing layer overestimated the values of $\overline{u_{1,1}^2}$ and $\overline{u_{1,1}^3}$ by about 15 and 21 % respectively (for $\sigma_{u_1}/U_1 \simeq \sigma_{u_2}/U_1 \simeq \sigma_{u_3}/U_1 \simeq 0{\cdot}17$), the skewness ($\equiv \overline{u_{1,t}^3}/\overline{u_{1,t}^2}^{\frac{3}{2}}$) and flatness factor ($\equiv \overline{u_{1,t}^4}/\overline{u_{1,t}^2}^2$) of $u_{1,t}$ agreed, to within 2 %, with the skewness and flatness factor of $u_{1,1}$ obtained from simplified relations using either the Lumley or Heskestad approach.

In the present paper, we concentrate on evaluating the shape of the probability density function of $\alpha_{,t}$ ($\alpha$ stands for either $u_1$ or $\theta$) when the probability density function of $\alpha_{,1}$ is assumed to be Gaussian. Two cases are considered in § 3 for the probability density function of the convection velocity. In the first, the fluctuating convection velocity $U_1 + u_1$ is assumed to be Gaussian while, in the second, the convection velocity is assumed equal to $(v_i v_i)^{\frac{1}{2}}$ where the fluctuations $v_i = \delta_{i1} U_1 + u_i$ are Gaussian. Measured probability density functions of $U_1^{-1}\alpha_{,t}$ in the atmospheric surface layer are compared in § 4 with those of $(U_1 + u_1)^{-1}\alpha_{,t}$ to ascertain, from an experimental viewpoint, the influence of a fluctuating convection velocity on the probability density

† Lumley's model was also extended by Chock (1978) to predict the effect of a fluctuating convection velocity on the eddy convection velocity for the high-frequency region of velocity and scalar spectra.

function of $\alpha_{,t}$. We show that the experimental trend is consistent with an alternative approach where $\alpha_{,t}$ and $v_1$ are assumed to be independent.

## 2. Analytical considerations

The equation for the velocity fluctuation $u_i$ may be obtained by subtracting the Reynolds equation from the Navier–Stokes equation and may be written as (e.g. Lumley & Panofsky 1964)

$$u_{i,t} = -u_{i,1}U_1 - u_{i,j}u_j - \delta_{i1}U_{1,j}u_j - \overline{u_{i,j}u_j} - p_{,i} + \nu u_{i,jj}, \tag{2}$$

where repeated indices are summed, $p$ is the kinematic pressure fluctuation and it has been assumed that there are no mean-flow velocity components in the $x_2$ and $x_3$ directions. The equation for $\theta$ (obtained by subtracting the mean heat-transfer rate equation from the instantaneous heat-transfer equation) may be written as

$$\theta_{,t} = -\theta_{,1}U_1 - u_j\theta_{,j} - u_jT_{,j} + \overline{u_j\theta_{,j}} + \mathscr{D}\theta_{,jj}, \tag{3}$$

where $\mathscr{D}$ is the thermal diffusivity and $T$ is the mean temperature. The last term on the right-hand side of (2) or (3) may be neglected for Reynolds or Péclet numbers that are sufficiently large (such as those found in the atmospheric surface layer). Using an order-of-magnitude argument, it may be shown, for sufficiently large Reynolds and Péclet numbers, that the third term may be neglected vis-à-vis the first term while the correlation term is zero if independence between large and small scales is invoked (e.g. Heskestad 1965). The third and fourth terms are strictly zero for homogeneous turbulence. It follows from the previous remarks that (3) can be reduced to

$$\theta_{,t} = -\theta_{,1}U_1 - u_j\theta_{,j} \tag{4}$$

while (2) becomes

$$u_{i,t} = -u_{i,1}U_1 - u_ju_{i,j} \tag{5}$$

only when the pressure term is ignored. In ignoring this term, Heskestad used Batchelor's (1951) estimate, for an isotropic field, of the ratio of a component of the mean-square pressure gradient to a typical mean-square inertia term, i.e.

$$\overline{p_{,1}^2}/(\overline{u_1^2}\,\overline{u_{1,1}^2}) = (0{\cdot}11R_\lambda^{\frac{1}{2}})^{-2}.$$

For $R_\lambda = 5000$, typical of the atmospheric boundary layer and sufficiently large to ignore the viscous term in (2) and to justify the assumption of isotropy, this ratio is 0·016. The ratio of the pressure gradient to an inertia term in (2) should be about 0·13 (for $R_\lambda = 5000$), which seems sufficiently small for the effect of pressure gradient to be considered negligible. As no pressure term appears in (3), Taylor's hypothesis (1) may be thought to be a better approximation to (4) than to (5). Strictly, (4) and (5) reduce to (1) for unusually small turbulence intensities.

The relations between moments of $\theta_{,t}$ and $\theta_{,1}$ may be readily written (corresponding relations for velocity derivatives have already been given by Heskestad and others) when the spatial temperature gradient $\theta_{,i}$ is assumed to be isotropic

$$\overline{\theta}_{,t} = -U_1\overline{\theta}_{,1} = 0,$$

$$\overline{\theta_{,t}^2} = \overline{v_iv_j\theta_{,i}\theta_{,j}}$$

or, equivalently,

$$\overline{\theta_{,t}^{2}} = \overline{\theta_{,1}^{2}}\,(U_1^2+\overline{u_1^2}+\overline{u_2^2}+\overline{u_3^2}) \tag{6}$$

(since $\overline{\theta_{,i}\theta_{,j}} = \overline{\theta_{,1}^{2}}\,\delta_{ij}$ by isotropy),

$$\overline{\theta_{,t}^{3}} = -(3\overline{u_1^2}+U_1^2)\,U_1\overline{\theta_{,1}^{3}} = 0, \tag{7}$$

$$\overline{\theta_{,t}^{4}} = \overline{\theta_{,1}^{4}}\,(3\overline{u_1^2}^2+3\overline{u_2^2}^2+3\overline{u_3^2}^2+6u_1^2U_1^2+U_1^4). \tag{8}$$

Equation (6) is in agreement with that given by Champagne *et al.* (1977) and Wyngaard & Clifford (1978). If the Reynolds number is sufficiently large, the assumption that $\theta_{,i}$ is isotropic may be relaxed to an assumption of local isotropy since most of the contribution to the $\theta_{,i}$ spectrum should then occur at relatively large wavenumbers.

For $\sigma_{u_1}/U_1 \simeq \sigma_{u_2}/U_1 \simeq \sigma_{u_3}/U_1 = 0{\cdot}17$, (6) and (8) indicate that $\overline{\theta_{,1}^{2}}$ and $\overline{\theta_{,1}^{4}}$ are overestimated by about 9 and 18 % respectively. However, the flatness factor of $\theta_{,t}$ correctly predicts the flatness factor of $\theta_{,1}$, which is essentially in agreement with the velocity derivative result of Wyngaard & Tennekes (1970).

## 3. Calculated probability density functions of $\theta_{,1}$

With the oversimplifying assumption that velocity fluctuations $u_2$ and $u_3$ are ignored, (4) and (5) may be further reduced to

$$\theta_{,t} = -\theta_{,1}(U_1+u_1), \tag{9}$$

$$u_{1,t} = -u_{,1}(U_1+u_1). \tag{10}$$

If it is also assumed that both $u_1$ and $\alpha_{,1}$ are Gaussian, the probability density function (p.d.f.) of $\alpha_{,t}$ may simply be obtained as the p.d.f. of the product of two random Gaussian variables. While the assumption of a Gaussian p.d.f. for $u_1$ is, usually, not unreasonable, the assumptions that $\theta_{,1}$ and $u_{1,1}$ are Gaussian are more difficult to justify. We cannot readily resort to experimental data for $\alpha_{,1}$ as the determination of $\alpha_{,1}$ from the difference between two sensors (velocity or temperature) separated in the $x_1$ direction is no doubt affected by the possibility of flow interference when the separation between sensors is small.

Batchelor (1953) observed that the skewness of the difference $u_1'-u_1$, where $u_1' = u_1(r+r_1)$, measured by Stewart (1961) in grid turbulence, is not inconsistent, when $r_1$ approaches zero, with the measured skewness of $u_{1,t}$. It was suggested that this evidence supported Taylor's hypothesis or the approximation that variations in velocity at a point fixed relative to the grid are the same as if the turbulence were simply convected by the stream. When $r_1 \to 0$, $u_1'-u_1$ should have a probability distribution governed by the Navier–Stokes equation. Batchelor argues that 'eddies smaller than a certain size owe their existence entirely to the non-linear transfer down the spectrum, and the smaller the eddy the more prolonged, so to speak, has been the influence of the non-linear terms'. The p.d.f. of $u_1'-u_1$ should therefore depart furthest from a normal distribution at very small values of $r_1$. It is well established that, for locally isotropic turbulence, the skewness of $u_{1,1}$ cannot be zero as it appears as a factor for the production term in the equation for the mean squared fluctuating vorticity. The p.d.f. of $u_{1,t}$ measured by Townsend (see Batchelor 1953,

p. 173) in grid turbulence is consistent with the expected skewness of $u_{1,1}$. Deviations of this p.d.f. from the Gaussian distribution are not significant however, probably because of the relatively low Reynolds number of the experiment.

Local isotropy requires that odd-order moments of $\theta_{,1}$ are zero, consistent with the assumption of a Gaussian probability density function. Measured non-zero values of $\overline{\theta_{,t}^3}$ cannot, in any case (e.g. Sreenivasan & Antonia 1977) unequivocally be interpreted as a violation of local isotropy because of the likely contribution to $\overline{\theta_{,t}^3}$ by the large-scale structure. The assumption of a symmetrical p.d.f. for $\alpha_{,1}$ would, in principle, seem more adequate for $\theta_{,1}$ than for $u_{1,1}$. Experimental evidence (§ 4) indicates, however, that normalized high even-order moments of $\theta_{,1}$ are much larger, especially at large Reynolds numbers, than corresponding moments of $u_{1,1}$, so that the assumption of a Gaussian p.d.f. for $\theta_{,1}$ is unlikely to be adequate. In particular, the assertion by Townsend (1959) that the experimental evidence suggests that the departure from normality of $\theta_{,1}$ may not be large cannot be reconciled with our previous observations.

With the additional assumption of independence between $\alpha_{,1}$ and $v_1$ and working in terms of the normalized variables

$$x = \alpha_{,t}/\sigma_{\alpha_{,t}},$$
$$x_{u_1} = (v_1 - U_1)/\sigma_{u_1},$$
$$x_{\alpha_{,1}} = \alpha_{,1}/\sigma_{\alpha_{,1}},$$

it can be shown that

$$x = -x_{\alpha_{,1}}(x_{u_1} + \xi)/(1+\xi^2)^{\frac{1}{2}}, \tag{11}$$

where $\xi = U_1/\sigma_{u_1}$ is the reciprocal of the turbulence intensity. It follows that the odd-order moments of $x$ vanish while the even-order moments (except $\overline{x^2}$) depend on the turbulence intensity level. For instance, the first four even-order moments are

$$\overline{x^2} = 1,$$

$$\overline{x^4} = \frac{\overline{x^4_{\alpha,1}}\,(\overline{x^4_{u_1}} + 6\xi^2 + \xi^4)}{(1+\xi^2)^2} = \frac{3(3+6\xi^2+\xi^4)}{(1+\xi^2)^2},$$

$$\overline{x^6} = \frac{\overline{x^6_{\alpha,1}}\,(\overline{x^6_{u_1}} + 15\xi^4 + 15\overline{x^4_{u_1}}\,\xi^2 + \xi^6)}{(1+\xi^2)^3} = \frac{15(15+15\xi^4+45\xi^2+\xi^6)}{(1+\xi^2)^3},$$

$$\overline{x^8} = \frac{\overline{x^8_{\alpha,1}}\,(\overline{x^8_{u_1}} + 28\overline{x^6_{u_1}}\,\xi^2 + 70\overline{x^4_{u_1}}\,\xi^4 + 28\overline{x^2_{u_1}}\,\xi^6 + 8\xi^8)}{(1+\xi^2)^4} = \frac{105(105+420\xi^2+210\xi^4+28\xi^6+\xi^8)}{(1+\xi^2)^4}.$$

Note that $\overline{x^{2n}}$ varies from $\overline{x_G^{2n}}$ to $(\overline{x_G^{2n}})^2$ as $\xi$ varies from $\infty$ to zero, where $\overline{x_G^{2n}}$ is the corresponding Gaussian value so that, for moderate turbulence intensity levels, the departure from the Gaussian values of $\overline{x^{2n}}$ is significant. For a 10 % turbulence level

$\overline{x^{2n}}/\overline{x_G^{2n}}$ is 0·04, 0·12 and 0·25 for $n = 2$, 3 and 4. This also reflects itself in the shape of the p.d.f. of $x$ for moderate values of $\xi$ as seen below.

Also, it is easy to establish that the p.d.f. of $\theta_{,t}$ or $u_{1,t}$ (9 and 10) will not deviate significantly from the p.d.f. of $\theta_{,1}$ or $u_{1,1}$ when $\sigma_{u_1} \ll U_1$. The following perturbation scheme

$$\theta_{,1} = \epsilon\theta_{,1}^{(1)} + O(\epsilon^2),$$
$$v_1 = U_1 + \epsilon u_1 + O(\epsilon^2),$$
$$\theta_{,t} = \epsilon\theta_{,t}^{(1)} + \epsilon^2\theta_{,t}^{(2)} + O(\epsilon^3),$$

where $\epsilon = O(\sigma_{u_1}/U_1)$, suggests that

$$\theta_{,t}^{(1)} = U_1\theta_{,1}^{(1)}, \quad \theta_{,t}^{(2)} = u_1\theta_{,1}^{(1)}, \quad \dots$$

while $\theta_{,t}^{(1)}$ follows the distribution of $\theta_{,1}^{(1)}$ ($\equiv (\theta_{,1})$, $\theta_{,t}^{(2)}$ follows $K_0$,† the zeroth-order modified Bessel function of the second kind, if $u_1$ and $\theta_{,x}^{(1)}$ are Gaussianly distributed. The probability density function of $\theta_{,t}$ is then given by

$$p_{\theta,t} = p_{\epsilon\theta_{,t}^{(1)}} * p_{\epsilon^2\theta_{,t}^{(1)}}$$

where $p$ denotes the p.d.f. and $*$ denotes convolution. The fluctuation $u_1$ only affects $p_{\theta,t}$ through terms of $O(\epsilon^2)$. When $\theta_{,1}$ and $(U_1 + u_1)$ are Gaussian, the convolution should follow closely a Gaussian p.d.f. since $K_0$ is mainly concentrated at the origin. The addition of the $u_2$ and $u_3$ terms in (9) and (10) should not affect this result.

To quantify the effect of a fluctuating convection velocity on the p.d.f. of $\alpha_{,t}$, it seems worthwhile to calculate $p_{\alpha,t}$ with the, admittedly crude, assumption that $p_{\alpha,1}$ is Gaussian. With the further assumption that the Gaussian fluctuations $u_1$ and $\alpha_{,1}$ are statistically independent, $p_{\alpha,t}$ is written as

$$p_{\alpha,t} = \int_{-\infty}^{\infty} p_{u_1}(v_1)\, p_{\alpha,1}\left(\frac{\alpha_{,t}}{v_1}\right)\frac{dv_1}{|v_1|} \tag{12}$$

with

$$p_{v_1} = \frac{1}{(2\pi)^{\frac{1}{2}}\sigma_{u_1}} \exp\left\{-\frac{(v_1 - U_1)^2}{2\sigma_{u_1}^2}\right\} \tag{13}$$

and

$$p_{\alpha,1} = \frac{1}{(2\pi)^{\frac{1}{2}}\sigma_{\alpha,1}} \exp\left(-\frac{\alpha_{,1}^2}{2\sigma_{\alpha,1}^2}\right). \tag{14}$$

It is not difficult to show that (12) reduces to

$$p_{\alpha,t} = \frac{e^{-\frac{1}{2}\xi^2}}{\pi\sigma_{u1}\sigma_{\alpha,1}} \sum_{k=0}^{\infty} \frac{\xi^{2k}}{(2k)!}\eta^k K_k(|\eta|), \tag{15}$$

where

$$\eta = \frac{\alpha_{,t}}{\sigma_{u_1}\sigma_{\alpha,1}},$$

and $K_k(x)$ is the modified Bessel function of the second kind.

This may be rewritten in normalized form as

$$p_x = \frac{1}{\pi}(1+\xi^2)^{\frac{1}{2}} e^{-\frac{1}{2}\xi^2} \sum_{k=0}^{\infty} \frac{\xi^{2k}}{(2k)!}\eta^k K_k(|\eta|), \tag{16}$$

† Proof that the distribution of the product of two Gaussian variables follows $K_0$ may be found in Antonia & Luxton (1971) and Lu & Willmarth (1972).

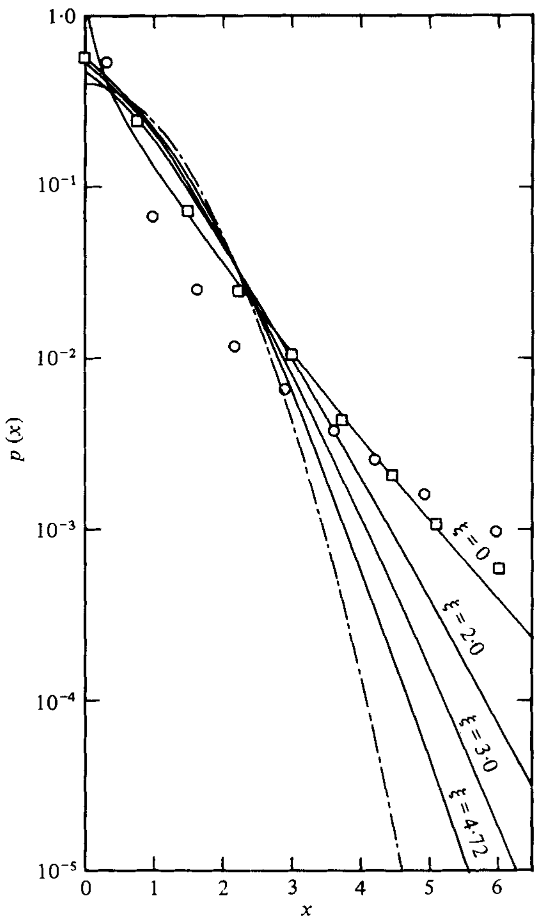


FIGURE 1. Probability density function of $x$ calculated from (16) for $\xi = 0$, 2·0, 3·0 and 4·72. □, $p_{u_{1,t}}$; ○, $p_{\theta,t}$; ———, Gaussian.

where $x = \eta/(1+\xi^2)^{\frac{1}{2}}$. Near $\eta = 0$, $\eta^k K_k(\eta) \simeq (k-1)!\,2^{k-1}$ $(k \neq 0)$ so that

$$p_x = \pi^{-1}(1+\xi^2)^{\frac{1}{2}}\,e^{-\frac{1}{2}\xi^2}\left\{-\ln(|\eta|/2)-\gamma+\sum_{k=1}^{\infty}\frac{2^{k-1}\,\xi^{2k}}{k(k+1)\dots 2k}\right\}, \tag{17}$$

where $\gamma$ is the Euler constant. When $U_1 = 0$, (15) simplifies to $p_x = \pi^{-1}K_0(|\eta|)$, a result which has been used in the literature (e.g. Antonia & Atkinson 1973) for comparison with measurements of the p.d.f. of $u_1u_2$ (or $u_2\theta$).

Distributions of $p_x$ for several values of $\xi$, including $\xi = 0$, are shown in figures 1 and 2. For comparison, also shown in these figures are the Gaussian distribution and experimental values of $p_{\alpha_{,t}}$ obtained in the atmospheric surface layer (details are given in § 4). While $p_x$ deviates only slightly from the Gaussian distribution at large $\xi$, the deviation at small values of $\xi$ becomes significant as $x$ increases. For $\xi = 3$, typical of the turbulence intensity level found on the axis of a circular jet, $p(x)$ exceeds the Gaussian value by almost one order of magnitude at four standard deviations. The experimental data (only positive values of $u_{1,t}$ and $\theta_{,t}$ are shown in figure 1)

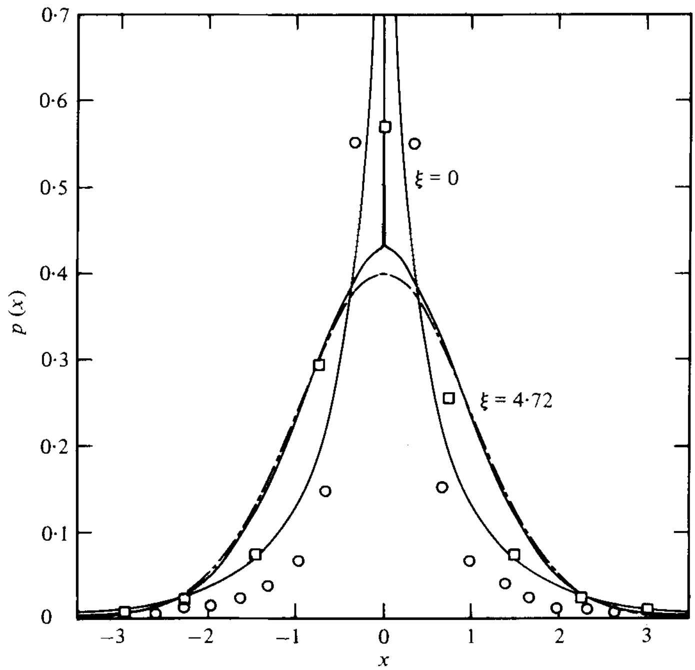


FIGURE 2. Probability density function of $x$ calculated from (16) for $\xi = 0$ and 4·72 showing detail for $|x| < 3{\cdot}5$. □, $p_{u_{1,t}}$; ○, $p_{\theta,t}$; ———, Gaussian.

correspond to $\xi = 4{\cdot}72$ and are clearly not well represented by $p_x$ ($\xi = 4{\cdot}72$). The poor agreement may be interpreted to reflect the inadequacy of (14) and of the assumed independence of $p_{v_1}$ and $p_{\alpha_{,1}}$. It is not difficult to show that if the p.d.f. of $\alpha_{,1}$ exhibits a spiky structure, this structure will be preserved in the p.d.f. of $\alpha_{,t}$. Suppose $p_{\alpha_{,1}}$ can be represented by a delta sequence $\delta_k$ (Greenberg 1971):

$$p_{\alpha_{,1}}(\alpha_{,1}) = \delta_k(\alpha_{,1}),$$

then the p.d.f. of $\alpha_{,t}$ is given by (12)

$$p_{\alpha_{,t}}(w;k) = \int_{-\infty}^{\infty} \delta_k(u)\, p_{u_1}\left(\frac{w}{u}\right)\frac{du}{|u|}.$$

Since, for any arbitrary $h(w)$,

$$\lim_{k\to\infty}\int_{-\infty}^{\infty} p_{\alpha_{,t}}(w;k)\, h(w)\, dw = \lim_{k\to\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta_k(u)\, p_{u_1}\left(\frac{w}{u}\right) h(w)\,\frac{du}{|u|}\, dw$$
$$= \int_{-\infty}^{\infty} \delta(w)\, h(w)\, dw = h(0),$$

$p_{\alpha_{,t}}(w;k)$ is also a delta sequence and the spiky structure in $p_{\alpha_{,1}}$ is also present in $p_{\alpha_{,t}}$. This, in addition to the experimental evidence, suggests that $p_{\alpha_{,1}}$ may be better presented by a spiky p.d.f. $K_0$ may be, perhaps speculatively, a better representation of $p_{\alpha,_1}$, or at least $p_{\theta,_1}$, than the normal distribution.

Townsend (1959) calculated $p_{\theta_{,t}}$ with $\theta_{,t}$ assumed to be given by

$$\theta_{,t} = -q\theta_{,1},$$

where $q = (v_i v_i)^{\frac{1}{2}}$ (no mean flow velocity was considered so that $v_i$ and $u_i$ are identical) and fluctuations $v_i$ and $\theta_{,1}$ were assumed to be Gaussian and independent. This calculation can be extended to the case where the flow has a non-zero mean velocity with a Gaussian joint p.d.f. of $v_i$ given by

$$p_{v_1 u_2 u_3} = \left(\frac{1}{2\pi\sigma_q}\right)^{\frac{3}{2}} \exp\left\{-\frac{(v_1 - U_1)^2 + u_2^2 + u_3^2}{2\sigma_q^2}\right\}.$$

The p.d.f. of $q \equiv (v_1^2 + u_2^2 + u_3^2)^{\frac{1}{2}}$ is then given by

$$p_q = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{q}{\sigma_q U_1} \sinh\left(\frac{qU_1}{\sigma_q^2}\right) \exp\left\{-\frac{q^2 + U_1^2}{2\sigma_q^2}\right\}. \tag{18}$$

When $U_1 = 0$, this reduces to the Maxwellian distribution

$$p_q = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{q^2}{\sigma_q^3} \exp\left\{-\frac{q^2}{2\sigma_q^2}\right\}$$

considered by Townsend (1959).

The p.d.f. of $\alpha_{,t} = -q\alpha_{,1}$ may be written as, when $p_{\alpha_{,1}}$ is Gaussian and $p_q$ is given by (18),

$$p_{\alpha_{,t}} = \frac{\exp[-U_1^2/2\sigma_q^2]}{\pi\sigma_q\sigma_{\alpha,1}} \sum_{k=0}^{\infty} \frac{(U_1/\sigma_q)^{2k}}{(2k+1)!} \left(\frac{\alpha_{,t}}{\sigma_q\sigma_{\alpha_{,1}}}\right)^{k+1} K_{k+1}\left(\frac{|\alpha_{,t}|}{\sigma_q\sigma_{\alpha_{,1}}}\right), \tag{19}$$

or, equivalently, in the normalized form

$$p_x = \pi^{-1} e^{-\frac{1}{2}\xi^2} (1+\xi^2)^{\frac{1}{2}} \sum_{k=0}^{\infty} \frac{\xi^{2k}}{(2k+1)!} \eta^{k+1} K_{k+1}(|\eta|), \tag{20}$$

where $\eta$ and $\xi$ are now defined by

$$\xi = U_1/\sigma_q, \quad \eta = \alpha_{,t}/\sigma_q\sigma_{\alpha_{,1}}$$

and $x = \eta/(1+\xi^2)^{\frac{1}{2}}$ is the normalized variable. Note that when $U_1 = 0$, (20) reduces to

$$p_x = \frac{1}{\pi} x K_1(|x|),$$

which is identical to the expression given by Townsend (for $\theta_{,t} \geqslant 0$)

$$-\tfrac{1}{2}\theta_{,t} H_1^{(1)}(i\theta_{,t}),\dagger$$

where

$$H_1^{(1)}(ix) = -\frac{2}{\pi} K_1(x)$$

is the first-order Hankel function of the first kind.

Also, since

$$\int_0^\infty x^k K_l(x)\, dx = 2^{k-1} \Gamma\left(\frac{1+k+l}{2}\right) \Gamma\left(\frac{1+k-l}{2}\right) \quad (\mathrm{Re}\,(k+1 \pm l) > 0),$$

it is easily verified that (16) and (20) satisfy the normalization conditions

$$\int p_x\, dx = 1 \quad \text{and} \quad \int x^2 p_x\, dx = 1.$$

† While values given in table 4 of Townsend (1959, appendix), are correct, those shown in figure 24 are incorrect.

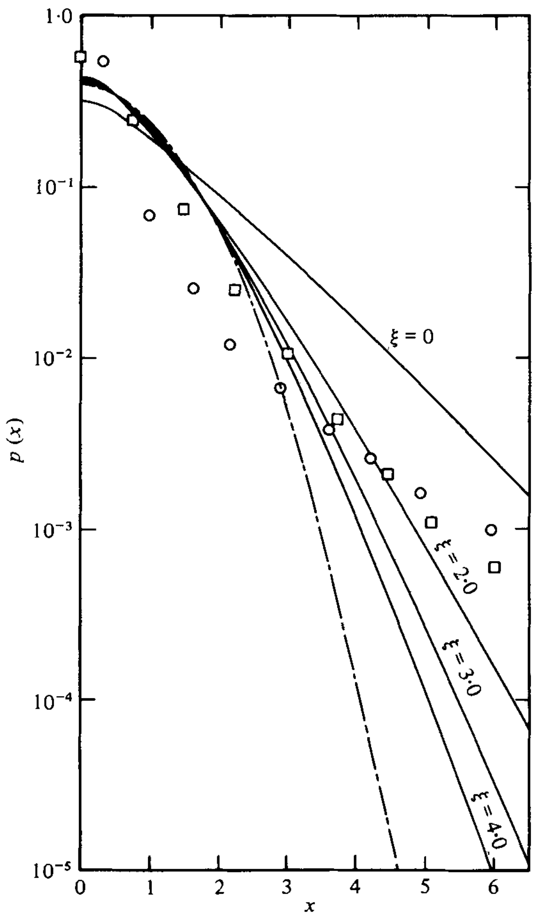


FIGURE 3. Probability density function of $x$ calculated from (20) for $\xi = 0$, 2·0, 3·0 and 4·0. □, $p_{u_1,t}$; ○, $p_{\theta,t}$; — — —, Gaussian.

Distributions of $p_x$, given by (20), are shown in figure 3 for different values of $\xi = U_1/\sigma_q$. (The experimental values of $p_{u_1,t}$ and $p_{\theta,t}$ correspond to $\xi \simeq 3$). Deviations of $p_x$ from the Gaussian curve are qualitatively similar to those of figure 1 for decreasing $\xi$. The agreement, in figure 3, between experimental values and (20) is, however, better than the corresponding agreement in figure 1. An expansion of (20) near $|\eta| = 0$ leads to

$$p_x = \pi^{-1}(1+\xi^2)^{\frac{1}{2}} e^{-\frac{1}{2}\xi^2} \sum_{k=0}^{\infty} \frac{2^k k!}{(2k+1)!} \xi^{2k}, \tag{21}$$

which is markedly different from (17), especially for low values of $\xi$ where the effect of the log singularity in (17) is more pronounced.

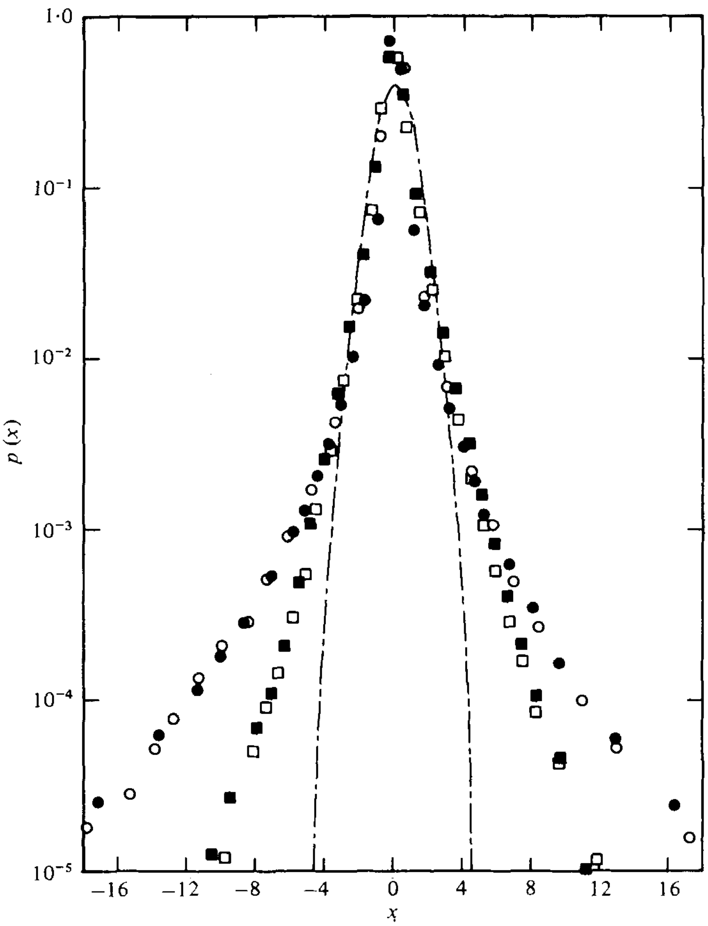


FIGURE 4. Experimental probability density functions of $\beta_1$, $\beta_2$, $\gamma_1$ and $\gamma_2$. □, $U_1^{-1}u_{1,t}$; ■, $(U_1+u_1)^{-1}u_{1,t}$; ○, $U_1^{-1}\theta_{,t}$; ●, $(U_1+u_1)^{-1}\theta_{,t}$; ————, Gaussian.

## 4. Experimental probability density functions of $u_{1,t}$ and $\theta_{,t}$ and discussion

Velocity $u_1$ and temperature $\theta$ fluctuations were measured at a height of 4 m in the atmospheric surface layer over a ploughed surface at the Bungendore field site of the C.S.I.R.O. Division of Environmental Mechanics. The fetch was about 300 m in the prevailing wind direction. The mean velocity profile, measured with Cassella cup anemometers at heights of 0·5, 1, 2, 4 and 8 m above ground, indicated a roughness length of about 1 mm. Fluctuations $u_1$ and $\theta$ were measured with a single hot wire and a cold wire respectively, both about 1 mm long, placed in a vertical direction. The distance between the parallel wires was approximately 1 mm.†

For $u_1$, a 5 mm diameter Wollaston wire was operated at an overheat ratio of 1·8 by a nonlinearized DISA 55 M01 constant-temperature anemometer. The fluctuation $\theta$ was measured with a Pt/10 % Rh Wollaston 'cold' wire of 0·6 $\mu$m diameter, operated with a d.c. constant-current bridge at a current of 0·1 mA. This low value of heating

† Details of both experimental site and conditions may be obtained in Antonia *et al.* (1979).

current ensured that the velocity sensitivity of the cold wire was negligible for existing experimental conditions. The temperature sensitivity of the hot wire was negligible. Signals from the anemometers were passed through buck and gain units before recording on a four-channel Hewlett–Packard FM tape recorder (model 3964A) at a speed of 38·1 cm s$^{-1}$. The tape was later played back in the laboratory and the fluctuating voltages were filtered prior to digitization on a PDP 11/20 computer. The cut-off ($-3$ dB) frequency of the filters was set equal to the Kolmogorov frequency $f_K$ ($= U_1/2\pi L_K$ with a mean wind speed $U_1 = 6{\cdot}1$ m s$^{-1}$ and Kolmogorov microscale $L_K$ of about 0·6 mm) while the sampling frequency was chosen equal to $2f_K$.

Signals proportional to $u_{1,t}$ and $\theta_{,t}$ were obtained by numerical differentiation. Probability density functions of these signals are shown in figure 4 for a record of about 12 min duration. Also shown in the figure are the p.d.f.'s of signals proportional to $(U_1+u_1)^{-1}u_{1,t}$ and $(U_1+u_1)^{-1}\theta_{,t}$. These latter quantities may be considered as rough approximations to $u_{1,1}$ and $\theta_{,1}$ respectively. The quantities $U_1^{-1}u_{1,t}$, $(U_1+u_1)^{-1}u_{1,t}$, $U_1^{-1}\theta_{,t}$, $(U_1+u_1)^{-1}\theta_{,t}$ are denoted by $\beta_1$, $\beta_2$, $\gamma_1$, $\gamma_2$ respectively for ease of reference in this section.

R.m.s. values of $\beta_2$ and $\gamma_2$ were found to be 16 % larger than the corresponding values of $\beta_1$ and $\gamma_1$. For the present data, $\sigma_{u_1}/U_1 = 0{\cdot}21$ while r.m.s. values of the components $u_2$ and $u_3$ were estimated from the experimental correlations (Bradley & Antonia 1978)

$$U_*^2/\sigma_{u_2}^2 \simeq 0{\cdot}021(-z/L)^{-\frac{2}{3}},$$
$$U_*^2/\sigma_{u_3}^2 = 0{\cdot}089+0{\cdot}171(-z/L)^{-\frac{2}{3}},$$

where $U_*$ is the friction velocity ($= 0{\cdot}26$ m s$^{-1}$) and $L$ is the Monin–Obukhov length ($z/L = -0{\cdot}50$). For values of $\sigma_{u_2}/U_1$ and $\sigma_{u_3}/U_1$ equal to 0·24 and 0·07 respectively, the relation (e.g. Heskestad 1965)

$$\overline{u_{1,t}^2} = U_1^2\overline{u_{1,1}^2}\left[1+\frac{\sigma_{u_1}^2}{U_1^2}+2\,\frac{\sigma_{u_2}^2+\sigma_{u_3}^2}{U_1^2}\right] \tag{22}$$

indicates that $\sigma_{\beta_2}$ should be 8 % lower than $\sigma_{\beta_1}$ while (6) indicates that $\sigma_{\gamma_2}$ should be about 5 % smaller than $\sigma_{\gamma_1}$. Note that (22) follows from (5) assuming independence between $u_j$ and $u_{i,j}$.

The effect of turbulence intensity on r.m.s. values of $\beta_2$ and $\gamma_2$ is in the opposite direction to that indicated by (6) and (22) when $\beta_2$ and $\gamma_2$ are identified with $u_{1,1}$ and $\theta_{,1}$ respectively. It is consistent, however, with the alternative formulation

$$\alpha_{,1} = -\frac{\alpha_{,t}}{v_1}, \tag{23}$$

where $\alpha_{,t}$ and $v_1$ are assumed to be independent. The p.d.f. of $\alpha_{,1}$ and its moments are then given by

$$p_{\alpha_{,1}}(\alpha_{,1}) = \frac{\sigma_u e^{-\frac{1}{2}\xi^2}}{\pi\sigma_{\alpha_{,t}}(1+\eta^2)}\left\{1+\left(\frac{\pi}{2}\right)^{\frac{1}{2}}\frac{\xi\, e^{\xi^2/2(1+\eta^2)}}{(1+\eta^2)^{\frac{1}{2}}}\,\phi\left(\frac{\xi}{(2(1+\eta^2))^{\frac{1}{2}}}\right)\right\},$$

where

$$\xi = U_1/\sigma_{u_1},$$
$$\eta = \alpha_{,1}\sigma_{u_1}/\sigma_{\alpha_{,t}},$$

and

$$\phi(x) = (2\pi)^{-\frac{1}{2}}\int_0^x e^{-t^2}\,dt.$$

It follows that

$$\overline{\alpha_{,1}^2} = \frac{\overline{\alpha_{,t}^2}}{U^2} \sum_{n=0}^{\infty} (2n+1) \frac{\overline{u_1^{2n}}}{U_1^{2n}}$$

$$= \frac{\overline{\alpha_{,t}^2}}{U^2}\left(1+3\frac{\overline{u_1^2}}{U_1^2}+5\frac{\overline{u_1^4}}{U_1^4}+\ldots\right), \tag{24}$$

$$\overline{\alpha_{,1}^4} = \frac{\overline{\alpha_{,t}^4}}{U_1^4}\left(1+10\frac{\overline{u_1^2}}{U_1^2}+35\frac{\overline{u_1^4}}{U_1^4}+\ldots\right), \tag{25}$$

so that a turbulence level of 20 % means an increase of 16 and 74 % for $\overline{\alpha_{,1}^2}$ and $\overline{\alpha_{,1}^4}$ over the 'uncorrected values' $\overline{\alpha_{,t}^2}/U_1^2$ and $\overline{\alpha_{,t}^4}/U_1^4$ respectively. When the correlation between $\alpha_{,t}$ and $v_1$ is included, a larger increase in $\overline{\alpha_{,1}^2}$ relative to $\overline{\alpha_{,t}^2}/U_1^2$ is expected since

$$\overline{\alpha_{,1}^2} = \frac{\overline{\alpha_{,t}^2}}{U_1^2} - 2\frac{\overline{u_1\alpha_{,t}^2}}{U_1^3} + 3\frac{\overline{u_1^2\alpha_{,t}^2}}{U_1^4}+\ldots$$

and physical considerations, similar to those that suggest that the presence of a large structure in the flow is mainly responsible for the non-zero skewness of $\theta_{,t}$, would indicate that $\overline{u_1\alpha_{,t}^2}$ is negative while $\overline{u_1^2\alpha_{,t}^2}$ is positive.

The qualitative agreement between (24), (25) and the trend of the data in figure 4 does not, of course, invalidate the earlier approach, as formulated by (9) and (10), with assumed independence between $\alpha_{,1}$ and $v_1$. As experimental values of the moments of $\alpha_{,1}$ are generated after dividing $\alpha_{,t}$ by $U_1+u_1$, it seems reasonable that the experimental trend qualitatively follows the predictions of (24) and (25). Equations (22) and (6) have been used, in the literature, to 'correct' measured values of $\overline{u_{1,t}^2}$ and $\overline{\theta_{,t}^2}$ respectively in order to obtain more meaningful estimates of the mean dissipation rates, assuming local isotropy, for $\overline{u_iu_i}$ and $\overline{\theta^2}$. These corrections differ from those of (24) since the final expression for the correction is sensitive to the type of operation – multiplication or division of the independent vaiiables – that is performed. The correct choice of operation is not *a priori* obvious. As little is known about the quantity $\alpha_{,1}$, the experimentalist may tend to favour the division since quantities $\alpha_{,t}$ and $(U_1+u_1)$ are experimentally readily measurable. On the other hand, formulations (4) and (5) may have the advantage that all three components of $U_j$ are readily taken into account, provided that Heskestad's simplifying assumptions are made. Formulation (23) could, in principle, be modified to include $u_2$ and $u_3$, at least when $\alpha \equiv \theta$. In practice, while $\theta_{,2}$ and $\theta_{,3}$ can be obtained relatively easily, the simultaneous measurements of $u_2$ and $u_3$ at effectively the same location where $\theta_{,2}$ and $\theta_{,3}$ are obtained would prove rather difficult. Before a definitive recommendation can be made on whether formulation (23) should replace formulation (9) or (10), an investigation of the assumptions of independence and, perhaps of less importance, of local isotropy would seem necessary. It has previously been pointed out that the existence of an organized large structure in the flow would result, probably irrespectively of $R_\lambda$, in non-zero values of $\overline{u_1\alpha_{,t}^2}$ and $\overline{u_1\alpha_{,t}^2}$. Statistics of the spatial derivatives of the temperature fluctuation may need to be interpreted in the context of the organized motion (e.g. Sreenivasan, Antonia & Danh 1977; Antonia & Van Atta 1980).

The shapes of $p_{\beta_1}$ and $p_{\beta_2}$ in figure 4 are consistent with the expected positive and negative skewness of $\beta_1$ and $\beta_2$ respectively. The skewness of $\beta_1$ is 0·50, smaller than

| Order of moment | $U_1^{-1}u_{1,t} \equiv \beta_1$ | $(U_1+u_1)^{-1}u_{1,1} \equiv \beta_2$ | $U_1^{-1}\theta_{,t} \equiv \gamma_1$ | $(U_1+u_1)^{-1}\theta_{,t} \equiv \gamma_2$ |
|---|---|---|---|---|
| 3 | 0·50 | 0·54 | −0·67 | −0·85 |
| 4 | 13·3 | 14·2 | 72·9 | 83·0 |
| 5 | 24·5 | 29·2 | −368 | −616 |
| 6 | 713 | 880 | $2\cdot5\times10^4$ | $3\cdot55\times10^4$ |
| 7 | $2\cdot04\times10^3$ | $3\cdot02\times10^3$ | $-3\cdot26\times10^5$ | $-6\cdot26\times10^5$ |
| 8 | $8\cdot13\times10^4$ | $1\cdot17\times10^5$ | $1\cdot68\times10^7$ | $2\cdot86\times10^7$ |

TABLE 1. Normalized moments of measured velocity and temperature derivatives.

the values obtained in the atmosphere (e.g. Champagne 1978) at Reynolds numbers comparable to the present Reynolds number ($R_\lambda \simeq 10730$). The skewness of $\gamma_1$ is $-0\cdot85$, in general agreement with the nearly constant (independent of $R_\lambda$) value found in the literature. Normalized moments, of order 3 to 8, of $\beta_1$, $\beta_2$, $\gamma_1$, $\gamma_2$ are shown in table 1. Moments of $\beta_2$ and $\gamma_2$ are larger than those of $\beta_1$ and $\gamma_1$, the difference being more pronounced for odd-order moments than for even-order moments. This difference between the normalized moments is, as noted previously, not consistent with the analysis of § 2. Equations (8) and (6) suggest that the flatness factor of $\theta_{,1}$ should be approximately 4 % lower than that $\theta_{,t}$. When components $u_2$ and $u_3$ are neglected, the flatness factor of $\theta_{,1}$ should be 16 % smaller than that of $\theta_{,t}$.

It seems useful to compare the present experimental data with those of Kholmyanskiy (1972) who measured statistics of $\beta_1$ and $\beta_2$† at a height of 13·5 m above a steppe. Except for the difference in the nature of the surface, experimental conditions ($z/L \simeq -0\cdot51$, $U_* = 0\cdot29$ m s$^{-1}$) of Kholmyanskiy are not dissimilar to the present ones. The r.m.s. value of $\beta_2$ was larger than that of $\beta_1$, in quantitative agreement with the present result, but the difference (35 %) obtained by Kholmyanskiy seems to be unusually large. Skewness and flatness factors of $\beta_2$ were appreciably smaller, by about 15 and 45 % respectively, than those of $\beta_1$. This difference is in the opposite direction and of larger magnitude than that obtained for the present data. As p.d.f.'s of $\beta_1$ and $\beta_2$ are not given by Kholmyanskiy, it is difficult to ascertain the accuracy of the normalized moments given in his table.

## 5. Summary of conclusions

Equations for velocity and temperature fluctuations in a turbulent flow at sufficiently large $R_\lambda$ only reduce to an expression of Taylor's hypothesis when certain oversimplifying assumptions are made. These assumptions appear to be more reasonable in the case of temperature than in the case of velocity as the pressure fluctuation does not appear in the equation for the temperature fluctuation. The tails of probability density functions of $u_{1,t}$ or $\theta_{,t}$, that are calculated by assuming that the convection velocity $U_1+u_1$ is Gaussian and that the spatial derivatives $u_{1,1}$ and $\theta_{,1}$ are Gaussian, are increasingly affected by the turbulence intensity level. A similar effect is observed when the convection velocity is assumed equal to $(U_1^2+u_1^2+u_2^2+u_3^2)^{\frac{1}{2}}$, where the fluctuations $u_i$ are assumed Gaussian. Normalized probability density

† $\beta_2$ was obtained by analogue differentiation of $\ln v_1$. Records of 5 min duration were used for statistical analysis.

functions and associated moments of temporal derivatives of velocity and temperature fluctuations measured in the atmospheric surface layer have been compared with those of spatial derivatives obtained by dividing the temporal derivatives by the instantaneous fluctuating velocity $U_1+u_1$. The normalized moments of the spatial derivatives are larger than those inferred from Taylor's hypothesis, the increase being more significant for odd-order moments than for even-order moments. Both direction and magnitude of this effect are different to those derived from the simplified equations (4) and (5) but are consistent with those derived from equation (23). Statistics of $\alpha_{,1}$, obtained by dividing $\alpha_{,t}$ by $U_1+u_1$, may be more attractive, from an experimental point of view, than those obtained by Heskestad and others. In particular, the continued application of corrections, provided by equations (6) and (22), does not seem fully justifiable. Further work is needed on the assumptions of local isotropy and of independence between small and large scales of motion.

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## REFERENCES

ANTONIA, R. A. & ATKINSON, J. D. 1973 High-order moments of Reynolds shear stress fluctuations. *J. Fluid Mech.* **58**, 581.

ANTONIA, R. A., CHAMBERS, A. J., PHONG-ANANT, D. & RAJAGOPALAN, S. 1979 Properties of spatial temperature derivatives in the atmospheric surface layer. *Boundary-Layer Met.* **17**, 101.

ANTONIA, R. A. & LUXTON, R. E. 1971 Some statistical properties of turbulence in smooth and rough wall boundary layers. *Charles Kolling Res. Lab., Dept of Mech. Engng, Univ. of Sydney, Rep.* TN F-31.

ANTONIA, R. A. & VAN ATTA, C. W. 1979 Skewness of spatial derivatives of temperature in a turbulent boundary layer. *Phys. Fluids* **22**, 2430.

BATCHELOR, G. K. 1951 Pressure fluctuations in isotropic turbulence. *Proc. Camb. Phil. Soc.* **47**, 359.

BATCHELOR, G. K. 1953 *The Theory of Homogeneous Turbulence.* Cambridge University Press.

BRADLEY, E. F. & ANTONIA, R. A. 1979 Structure parameters in the atmospheric surface layer. *Quart. J. Roy. Met. Soc.* **105**, 695.

CHAMPAGNE, F. H. 1978 The fine-scale structure of the turbulent velocity field. *J. Fluid Mech.* **86**, 67.

CHAMPAGNE, F. H., FRIEHE, C. A., LARUE, J. C. & WYNGAARD, J. C. 1977 Flux measurements, flux estimation techniques, and fine-scale turbulence measurements in the unstable surface layer over land. *J. Atmos. Sci.* **34**, 515.

CHOCK, D. P. 1978 On eddy convection velocity in the atmospheric boundary layer. *Boundary-Layer Met.* **14**, 397.

FISHER, M. J. & DAVIES, P. O. A. L. 1964 Correlation patterns in a non-frozen pattern of turbulence. *J. Fluid Mech.* **18**, 97.

GREENBERG, M. D. 1971 *Application of Green's Functions in Science and Engineering.* Prentice Hall.

HESKESTAD, G. 1965 A generalized Taylor hypothesis with application for high Reynolds number turbulent shear flows. *Trans. A.S.M.E.* E, *J. Appl. Mech.* **87**, 735.

KHOLMYANSKIY, M. Z. 1972 Measurements of turbulent microfluctuations of the wind-velocity derivative in the surface layer. *Izv. Atmos. & Ocean Phys.* **8**, 818.

LU, S-S. & WILLMARTH, W. W. 1972 The structure of the Reynolds stress in a turbulent boundary layer. *Dept. Aerospace Engng, Gas Dynamics Lab. Univ. of Michigan, Tech. Rep.* 021490-2-T.

LUMLEY, J. L. 1965 Interpretation of time spectra measured in high-intensity shear flows. *Phys. Fluids* **8**, 1056.

LUMLEY, J. L. & PANOFSKY, H. A. 1964 *The Structure of Atmospheric Turbulence*, p. 67. Wiley.

SREENIVASAN, K. R., ANTONIA, R. A. & DANH, H. Q. 1977 Temperature dissipation fluctuations in a turbulent boundary layer. *Phys. Fluids* **20**, 1238.

SREENIVASAN, K. R. & ANTONIA, R. A. 1977 Skewness of temperature derivatives in turbulent shear flows. *Phys. Fluids* **20**, 1986.

STEWART, R. W. 1951 Triple velocity correlations in isotropic turbulence. *Proc. Camb. Phil. Soc.* **47**, 146.

TOWNSEND, A. A. 1959 Temperature fluctuations over a heated horizontal surface. *J. Fluid Mech.* **5**, 209.

TOWNSEND, A. A. 1976 *The Structure of Turbulent Shear Flow*, 2nd edn. Cambridge University Press.

WYNGAARD, J. C. & CLIFFORD, S. F. 1977 Taylor's hypothesis and high frequency turbulence spectra. *J. Atmos. Sci.* **34**, 922.

WYNGAARD, J. C. & PAO, Y. H. 1972 Some measurements of the fine structure at large Reynolds number turbulence. In *Statistical Models and Turbulence*. Lecture notes in Physics, vol. 12, p. 384. Springer.

WYNGAARD, J. C. & TENNEKES, H. 1970 Measurements of the small-scale structure of turbulence at moderate Reynolds numbers. *Phys. Fluids* **13**, 1962.